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On Basic Concepts of the Quasiclassical Operator Approach

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Abstract

We present a derivation of the probability of bremsstrahlung from high-energy electrons in a screened Coulomb field using the quasiclassical operator method. It is shown that recent Zakharov's criticism is completely groundless and comes from misunderstanding of the method. We confirm all our published results.

1. Comment, recently published by Zakharov [1], shows that it is reasonable to discuss once more the basis of the quasiclassical operator (QO) method developed by authors in [2], [3]. The method was presented in detail in the book [4] and later in the books [5] and [6] (the latter is the enlarged English translation of the book [5]). Very clear description of the QO method is given in the popular textbook [7].

This paper is divided into two parts. In the first part we discuss in frame of the QO method the derivation of the basic formulae describing the radiation process paying special attention to the important details of the approach. In the second part we analyze Zakharov's claims step by step and show that all his pretensions are erroneous because of complete misunderstanding of the QO method.

In this paper we consider the applicability of the QO method to the problem of radiation from a ultrarelativistic particle at a potential scattering. In this process the emitted photon and the final electron are moving at a small angle to the initial electron momentum and the large angular momenta $l \gg 1$ contribute. In this situation the quasiclassical scattering theory is applicable. The very important result of the method is that recoil at radiation is incorporated into the theory in the *universal* form for any external field. There are two essentially different cases in application of the QO method. In the first case scattering in a potential can be considered in *classical* terms: phase shifts are large, there is a correspondence between the impact parameter and the momentum transfer. Therefore it is possible to use the version of the QO method where one can instead of operators substitute classical variables in the *coordinate* space in an expression for probability of the process (see, [2], Secs.9-15 in [4], Secs.2-5 in [5], [6], Sec.90 in [7]). In the second case, the process of scattering can't be described in classical terms. However, for scattering of ultrarelativistic particles where large angular momentum contributes, one can use the *quasiclassical* approximation for description of scattering including the situation where the phase shifts are small (see [3], Secs.9,16-20 in [4], Secs.2,7 in [5], [6], Sec.96 in [7]). Precisely this formulation of the method must be applied for consideration of radiation from ultrarelativistic particles at scattering on atoms in a media. In our recent papers devoted to the theory of Landau-Pomeranchuk-Migdal (LPM) effect [8]-[11] we use the mentioned formulation of the QO method. Therefore we will analyze it in detail below.

2.Because the LPM effect can be observed at very high energies, we can consider the case of complete screening, so that (we employ units such that $\hbar = c = 1$)

$$a_s \ll \frac{1}{q_{min}} = \frac{2\varepsilon(\varepsilon-\omega)}{\omega m^2} \equiv l_0, \tag{1}$$

where a_s is the screening radius ($a_s \simeq 111 Z^{-1/3}\lambda_c, \lambda_c = 1/m$), Z is the charge of a nucleus, q_{min} is the minimal momentum transfer which is longitudinal (with respect to the momentum of the initial electron $\mathbf{p}$), $\varepsilon(\omega)$ is the energy of the

initial electron (the emitted photon), m is the electron mass, l_0 is the radiation formation length for a small angle scattering on an isolated atom. Note that in frame of the QO method the radiation problem is solved for the case of an arbitrary screening , see Sec.18 in [4]. The impact parameters ϱ, contributing into the scattering cross section, are small comparing the formation length ($\varrho \leq a_s \ll l_0$) in a screened Coulomb potential. This means that the scattering of the ultrarelativistic particles (the virtual electron is close to the mass shell) takes place independently of radiation process (see, Sec.7 in [5], [6], Sec.96 in [7]). Thus, we can present the cross section of radiation as a product of the probability of emission of a photon with the momentum $\mathbf{k}$ at given momentum transfer $\mathbf{q}_\perp$ ($\mathbf{q}_\perp \mathbf{p} = 0$), and the cross section of scattering $d\sigma(\mathbf{q}_\perp)$ of a particle with the same momentum transfer $\mathbf{q}_\perp$:

$$d\sigma_\gamma = W_\gamma(\mathbf{q}_\perp, \mathbf{k}) d^3k d\sigma(\mathbf{q}_\perp). \tag{2}$$

We show below that in frame of the QO method the probability of radiation $W_\gamma(\mathbf{q}_\perp, \mathbf{k})$ is given by the *classical* trajectory of a particle in "the form of an angle" in the *momentum space*

$$\mathbf{p}(t) = \vartheta(-t)\mathbf{p} + \vartheta(t)(\mathbf{p} + \mathbf{q}_\perp), \tag{3}$$

while the cross section $d\sigma(\mathbf{q}_\perp)$ should be taken in the *eikonal* form.

3. The probability of photon emission from an electron in frame of the QO method has the form (see, Eqs.(16.7)-(16.12) in [4], Eqs.(7.1)-(7.12) in [5], [6], Eqs.(96.1)-(96.8) in [7])

$$dw = < i|M^+ M|i >, \tag{4}$$

where

$$M = \frac{e}{2\pi\sqrt{\omega}} \int\limits_{-\infty}^{\infty} R(t) \exp\left[i \int\limits_0^t \frac{kp(t')}{\varepsilon - \omega} dt' \right], \tag{5}$$

here $R(t) = R(\mathbf{p}(t))$, $R(\mathbf{p})$ is the matrix element for the free particles depending on the electron spin, $kp = \omega\mathcal{H} - \mathbf{kp}, \mathcal{H} = \sqrt{\mathbf{p}^2 + m^2}$, $|i>$ is the state vector of the initial particle at the time $t = 0$, and $\mathbf{p}(t)$ is the operator of momentum in the Heisenberg picture:

$$\mathbf{p}(t) = \exp(-iHt)\mathbf{p}\exp(iHt), \quad H = \mathcal{H} + V(\mathbf{r}), \tag{6}$$

where $V(\mathbf{r})$ is the potential of an atom.

We present the evolution operator as

$$\exp(-iHt) = \exp(-i\mathcal{H}t)N(t), \quad N(t) = \exp(i\mathcal{H}t)\exp(-i(\mathcal{H} + V)t). \tag{7}$$

Differentiating the last expression over the time we obtain

$$\frac{dN(t)}{dt} = -i\exp(i\mathcal{H}t)V(\mathbf{r})\exp(-i(\mathcal{H} + V)t) = -iV(\mathbf{r} + \mathbf{v}t)N(t), \mathbf{v} = \frac{\mathbf{p}}{\mathcal{H}}. \tag{8}$$

The solution of this differential equation for the initial condition $N(0)=1$ is

$$N(t) = \mathrm{T}\exp\left[-i\int\limits_0^t V(\mathbf{r}+\mathbf{v}t')dt'\right], \tag{9}$$

where T is the operator of the chronological product. Bearing in mind that the commutator

$$[r_i, v_j] = \frac{i}{\mathcal{H}}(\delta_{ij} - v_i v_j), \tag{10}$$

one can drop operator T of the chronological product within relativistic accuracy (i.e. with accuracy up to terms $\sim 1/\gamma$) and present operator $N(t)$ as

$$N(t) \simeq \exp\left[-i\int\limits_0^t V(\boldsymbol{\varrho}, z+t')dt'\right], \tag{11}$$

where the axis z is directed along the momentum of the initial particle. If the formation time of radiation is much longer than the characteristic time of the scattering, one can present the dependence of the operator $\mathbf{p}(t)$ on the time in Eq.(5) as

$$\mathbf{p}(t) = \vartheta(-t)\mathbf{p}(-\infty) + \vartheta(t)\mathbf{p}(\infty), \quad \mathbf{p}(\pm\infty) = N^+(\pm\infty)\mathbf{p}N(\pm\infty). \tag{12}$$

It should be pointed out that in the case when the scattering process is of non-classical character the operators $\mathbf{p}(-\infty)$ and $\mathbf{p}(\infty)$ are noncommutative among themselves and, generally speaking, one can't neglect their commutator. Let us mention also that using Eq.(11) one find

$$\mathbf{p}_\perp(-\infty) \simeq \mathbf{p}_\perp + \int\limits_{-\infty}^{z} \boldsymbol{\nabla}_\varrho V(\boldsymbol{\varrho}, z')dz', \quad \mathbf{p}_\perp(\infty) \simeq \mathbf{p}_\perp - \int\limits_{z}^{\infty} \boldsymbol{\nabla}_\varrho V(\boldsymbol{\varrho}, z')dz'. \tag{13}$$

These approximate expressions are written in the classical form when the entering operators commutate among themselves. Substituting the "trajectory" (12) in Eq.(5) we obtain (this is Eq.(7.7) in [5], [6])

$$M = \frac{ie(\varepsilon-\omega)}{2\pi\sqrt{\omega}}\left[\frac{R(\mathbf{p}(\infty))}{kp(\infty)} - \frac{R(\mathbf{p}(-\infty))}{kp(-\infty)}\right]. \tag{14}$$

The state $|i>$ in Eq.(4) denotes the wave function in the configuration space in the time $t=0$. However, in the scattering problem under consideration (motion of a particle in a local potential), the wave function are defined at $t\to\pm\infty$. Let us consider the interconnection between these wave functions. By definition of the state evolution, we have

$$\lim_{t\to\infty}\exp(-i\mathcal{H}t)|\infty> = \lim_{t\to\infty}\exp(-iHt)|0>,$$

$$|0> = N^+(\infty)|\infty> \equiv |\mathrm{out}> \simeq \exp\left[i\int\limits_z^\infty V(\boldsymbol{\varrho}, z')dz'\right]|\infty>. \tag{15}$$

For the state evolving from $t \to -\infty$ we have

$$|0>= N^{+}(-\infty)|-\infty>\equiv |\text{in}>\simeq \exp\left[-i\int\limits_{-\infty}^{z} V(\boldsymbol{\varrho}, z')dz'\right]|\infty> . \tag{16}$$

As a rule, the state at $t \to \pm\infty$ are the plane waves, i.e. they are the eigenstates of the momentum operator $\mathbf{p}$. However, one can use also another definition of these states, e.g. wave packets. Such definition was used in description of the bremsstrahlung at the collision of two high-energy beams with the restricted transverse dimensions [12]. If there is a summation over the set of states, i.e. they are the intermediate states, choice of the states is determined by a convenience of calculation. This circumstance was used in [16] for the derivation of connection between spectra of the bremsstrahlung and the pair creation in the theory exact in $Z\alpha$.

4.It follows from the above analysis that for the derivation of the differential cross section of the bremsstrahlung in Eqs.(2), (4), it is necessary to insert the projection operator $|f>< f|$, which corresponds "out" states of the plane wave with the momentum $\mathbf{p}_f$, between the operators M^+ and M, and to take "in" states with the momentum $\mathbf{p}_i$ as the states $|i>$. So, the initial state is the eigenvector of the operator $\mathbf{p}(-\infty)$ and the final state is the eigenvector of the operator $\mathbf{p}(\infty)$:

$$\begin{aligned}
&\mathbf{p}(-\infty)|i>= N^{+}(-\infty)\mathbf{p}N(-\infty)N^{+}(-\infty)|\mathbf{p}_i>= \mathbf{p}_i|i>, \\
&\mathbf{p}(\infty)|f>= N^{+}(\infty)\mathbf{p}N(\infty)N^{+}(\infty)|\mathbf{p}_f>= \mathbf{p}_f|f> .
\end{aligned} \tag{17}$$

Using (17) we deduce for the matrix element of the operator M (14)

$$M = \frac{ie(\varepsilon-\omega)}{2\pi\sqrt{\omega}}\left[\frac{R(\mathbf{p}_f)}{kp_f} - \frac{R(\mathbf{p}_i)}{kp_i}\right] < f|i> . \tag{18}$$

Using the definitions of "in" and "out" states Eqs.(15),(16) we have

$$\begin{aligned}
< f|i> =< \mathbf{p}_f|N(\infty)N^{+}(-\infty)|\mathbf{p}_i> =< \mathbf{p}_f|\mathrm{T}\exp\left[-i\int\limits_{-\infty}^{\infty} V(\mathbf{r}+\mathbf{v}t)dt\right]|\mathbf{p}_i> \\
\simeq< \mathbf{p}_f|\exp\left[-i\int\limits_{-\infty}^{\infty} V(\mathbf{r}+\mathbf{v}t)dt - \frac{1}{2}\int\limits_{-\infty}^{\infty} dt_2\int\limits_{-\infty}^{\infty} dt_1[V(t_2), V(t_1)]\right]|\mathbf{p}_i>,
\end{aligned} \tag{19}$$

where $V(t_1)$ means $V(\mathbf{r}+\mathbf{v}t_1)$, etc. We retain here only the first two terms in the decomposition of the T-product being restricted to the commutator

$$[V(t_2), V(t_1)] = -\frac{i}{\mathcal{H}}(t_2-t_1)\boldsymbol{\nabla}_{\perp}V(t_2)\boldsymbol{\nabla}_{\perp}V(t_1). \tag{20}$$

The first term in the decomposition of the T-product in Eq.(19) gives the scattering amplitude in the eikonal approximation, the second term gives the correction to the eikonal approximation:

$$< f|i >= \int d^2\varrho \exp\left[i\mathbf{q}_\perp\boldsymbol{\varrho} + i\chi(\boldsymbol{\varrho})\right] 2\pi\delta(p_{f\parallel} - p_{i\parallel}), \tag{21}$$

where

$$\begin{aligned}
&\chi(\boldsymbol{\varrho}) = \chi_0(\boldsymbol{\varrho}) + \chi_1(\boldsymbol{\varrho}), \quad \chi_0(\boldsymbol{\varrho}) = -\int\limits_{-\infty}^{\infty} V(\boldsymbol{\varrho}, z)dz \\
&\chi_1(\boldsymbol{\varrho}) = \frac{1}{2\varepsilon}\int\limits_{-\infty}^{\infty} dz_2 \int\limits_{-\infty}^{\infty} dz_1 \boldsymbol{\nabla}_\perp V(z_2)\boldsymbol{\nabla}_\perp V(z_1)(z_2 - z_1)\vartheta(z_2 - z_1).
\end{aligned} \tag{22}$$

In the centrally symmetric potential we have

$$\chi_0(\varrho) = -\int\limits_{-\infty}^{\infty} V(\sqrt{\varrho^2 + z^2})dz, \quad \chi_1(\varrho) = -\frac{\varrho^2}{\varepsilon}\int\limits_{-\infty}^{\infty} \frac{\partial V^2(\sqrt{\varrho^2 + z^2})}{d\varrho^2}dz \tag{23}$$

In the Coulomb field $\chi_1 = (Z\alpha)^2\pi/(2\varrho\varepsilon)$. The main contribution into corrections to the probability of the bremsstrahlung connected with the phase χ_1 gives the impact parameters $\varrho \sim \lambda_c = 1/m$, so that $\chi_1 \sim (Z\alpha)^2/\gamma$. The corresponding corrections $\sim 1/\gamma$ to the bremsstrahlung cross section were calculated by authors in [13].

We introduce now the notations $\mathbf{p}_i \equiv \mathbf{p},\ \mathbf{p}_f = \mathbf{p}' + \mathbf{k}$ where $\mathbf{p}'$ is the momentum of electron after photon emission. Then neglecting the terms of the order of $q_\parallel$ in the argument of the δ-function in Eq.(21) we have in the region $q_\perp \gg q_\parallel$ which contributes for the potential considered

$$\delta(p_{f\parallel} - p_{i\parallel}) \simeq \delta(\varepsilon' + \omega - \varepsilon), \quad \varepsilon' = \sqrt{\mathbf{p}'^2 + m^2}. \tag{24}$$

Using this relation one can express the propagator kp_f through the propagator kp' (up to terms of the order $1/\gamma^2$)

$$\begin{aligned}
kp_f &= \omega\sqrt{(\mathbf{p}' + \mathbf{k})^2 + m^2} - \mathbf{k}(\mathbf{p}' + \mathbf{k}) \\
&= \omega\sqrt{\varepsilon^2 - 2kp'} + kp' - \omega\varepsilon \simeq \omega\varepsilon\left(1 - \frac{kp'}{\varepsilon^2}\right) + kp' - \omega\varepsilon = \frac{\varepsilon'}{\varepsilon}kp'.
\end{aligned} \tag{25}$$

Our final result for the differential probability of radiation in Eq.(2) is therefore

$$W_\gamma(\mathbf{q}_\perp, \mathbf{k}) = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega}\left|\frac{\varepsilon R(\mathbf{p}' + \mathbf{k})}{kp'} - \frac{\varepsilon' R(\mathbf{p})}{kp}\right|^2 \tag{26}$$

The explicit expression for the probability $W_\gamma(\mathbf{q}_\perp, \mathbf{k})$ with regard for polarization and spin effects are given in [4].

5. In the above analysis we traced in detail the transition to the expressions calculated on the trajectory of particle in "the form of an angle" in the momentum space. Precisely these *definite* trajectories (taking into account the recoil at the radiation) determine the probability of photon emission

$$\begin{aligned} dw &= \frac{\alpha}{(2\pi)^2}\frac{d^3k}{\omega}\int dt_2 \int dt_1 R^*(t_2)R(t_1)\exp\left[-\frac{i\varepsilon}{\varepsilon'}\int\limits_{t_1}^{t_2} kv(t)dt\right] \\ &= W_\gamma d^3k, \quad v = \frac{p}{\varepsilon} = (1, \mathbf{v}), \quad \varepsilon' = \varepsilon - \omega. \end{aligned} \tag{27}$$

When the projectile is moving in a medium it scatters on atoms. In this case the probability should be averaged over all possible trajectories with the weight function defined by the cross section $d\sigma(\mathbf{q}_\perp)$. In the presence of a macroscopic external field there is the systematic variation of the mean velocity of a projectile. The probability of radiation in this case is given by the same Eq.(27). Both, the systematic variation of the mean velocity of the projectile as well as the fluctuations of it velocity due to scattering with respect to the mean value are taken into account with the aid of the distribution function. Thus, when the projectile is moving in a medium the probability of photon emission per unit time has the form (see, Eq.(20.31) in [4] and Eq.(2.4) in [14])

$$\begin{aligned} dw &= \left\langle \frac{dw}{dt} \right\rangle = \frac{\alpha}{(2\pi)^2}\frac{d^3k}{\omega}\mathrm{Re}\int\limits_0^\infty d\tau \exp\left(-i\frac{\varepsilon}{\varepsilon'}\omega\tau\right)\int d^3v \int d^3v' \\ &\times \int d^3r \int d^3r' \mathcal{L}(\boldsymbol{\vartheta}, \boldsymbol{\vartheta}')F_i(\mathbf{r}, \mathbf{v}, t)F_f(\mathbf{r}', \mathbf{v}', \tau; \mathbf{r}, \mathbf{v})\exp\left[i\frac{\varepsilon}{\varepsilon'}\mathbf{k}(\mathbf{r}' - \mathbf{r})\right]. \end{aligned} \tag{28}$$

We consider here the case of an infinitely thick target. The distribution functions F_i and F_f satisfy the kinetic equation (see, Eq.(20.32) in [4] and Eq.(2.5) in [14])):

$$\begin{aligned} &\frac{\partial F(\mathbf{r}, \mathbf{v}, t)}{\partial t} + \mathbf{v}\frac{\partial F(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{r}} + \mathbf{w}\frac{\partial F(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} \\ &= n\int \sigma(\mathbf{v}, \mathbf{v}')\left[F(\mathbf{r}, \mathbf{v}', t) - F(\mathbf{r}, \mathbf{v}, t)\right]d^3v', \end{aligned} \tag{29}$$

where n is the number density of atoms in the medium, and $\sigma(\mathbf{v}, \mathbf{v}')$ is the scattering cross section which in our case should be calculated in the eikonal approximation in the screened Coulomb potential. The combinations entering Eq.(27) are

$$\int\limits_{t_1}^{t_2} kv(t)dt = \int\limits_{t_1}^{t_2}(\omega - \mathbf{k}\mathbf{v}(t))dt = \omega\tau - \mathbf{k}(\mathbf{r}(t_2) - \mathbf{r}(t_1)) \rightarrow \omega\tau - \mathbf{k}(\mathbf{r}' - \mathbf{r}),$$

$$R^*(t_2)R(t_1) = \frac{1}{2\varepsilon'^2}\left[\frac{\omega^2 m^2}{\varepsilon^2} + (\varepsilon^2+\varepsilon'^2)\mathbf{v}_\perp(t_2)\mathbf{v}_\perp(t_1)\right]$$
$$\to \frac{1}{2\varepsilon'^2}\left[\frac{\omega^2 m^2}{\varepsilon^2} + (\varepsilon^2+\varepsilon'^2)\boldsymbol{\vartheta}\boldsymbol{\vartheta}'\right] \equiv \frac{1}{2}\mathcal{L}(\boldsymbol{\vartheta},\boldsymbol{\vartheta}'). \tag{30}$$

Just this approach was used in papers [8]-[11] in the derivation of the principal equations in the theory of the LPM effect.

Let us analyze specific Zakharov's claims in p.4 of [1] (given in quotation marks).

1. ".. Eq.(1) (this is our Eq.(27)) derived neglecting the variation of the field acting on electron...". This statement is wrong. The Eq.(27), is valid for any variation of the external field. Although we consider the case of slowly varying external field in the first derivation [2], nevertheless for $\gamma \gg 1$ (this is the only case we consider), this result can used for the Coulomb field (see Eqs.(10) and (11) in [3]). Later we show that Eq.(27), is valid for any field (see [5], [6]) including the field of the plane wave where the dependence on both coordinate and time is essential.

2. "..In the quantum regime the above correspondence between the scattering angle and the electron impact parameter in interaction with a medium consistent is lost. In this case one must take into account accurately the variation of the field acting on the electron in evaluating the radiation rate. Eq.(1) (obtained for a smooth field) does not make any sense in the quantum regime. It is well known that for a screened Coulomb potential the quasiclassical situation takes place at $Z\alpha \gg 1$. In real media, when $Z\alpha < 1$ we have essentially quantum regime. For this reason, it is evident that Eqs.(1) and (2) (Eq.(2) we will discuss below) are not justified for real media. The use of Eqs.(1) and (2) which are not valid in the quantum regime is the main conceptual error in the treatment of the LPM effect given in Refs.[11, 12]. (here this is Refs.[4] and [14])."

   A few errors accumulate here. First of all it is evident that Zakharov is not familiar with classical papers of Bethe and Maximon [15] (see also [16]-[18]) where cross sections of the bremsstrahlung and the pair creation in the Coulomb field for high energy region are calculated with the use of the quasiclassical wave functions. The results obtained in the mentioned papers are valid for any value of $Z\alpha$, for low Z they coincide with the Bethe-Heitler cross section, for $Z\alpha \sim 1$ the cross sections contain additional term which is the Coulomb correction. We want to stress that our approach has the same region of applicability as the Bethe-Maximon theory. This theory is developed in frame of Schrödinger picture, while we are developed approach in frame of Heisenberg (operator) picture. Because of this, starting with

some stage of calculation our results coincide with those of Bethe and Maximon or Olsen, Maximon, and Wergeland [17]. The case of a screened Coulomb field is considered in the last paper.

Second, as it is shown above, our approach is basically quantum (quasiclassical) and only the final result can be in many cases, expressed in classical terms as it was explained above (and in [2]-[7]). Moreover, in the papers [3], [7] the case of the Coulomb field was considered. For calculation of the probability of radiation in this field for $q \sim q_{min}$ one can't use the trajectory "in the form of an angle", the impact parameters $\varrho \sim l_0$ contribute (see Eq.(1)), and the cross section of radiation can't be presented in the form Eq.(2). But even in this case the Eqs.(4) and (5) are valid. The underlined claim is erroneous. In fact, the quantum-mechanical properties of scattering are relevant part of our theory.

3. point 2 "The authors of Refs.[11,12] (this is Refs.[4], [14] here) compensate the incorrectness of Eq.(1) (this is Eq.(27) here) by another error in the procedure of averaging over the electron trajectories. According to their logic the QO expression (1) (this is Eq.(27) here) must be averaged over all possible classical trajectories (and author say this)." The last statement has nothing common with our method. As it was shown above the Eq.(27) is valid in all cases when the motion of particle can be described by quasiclassical quantum mechanics. As was explained above, the averaging procedure appears only in the case when we have to consider the scattering of the projectile in a medium. In this situation the radiation process is described by Eq.(28). In a very definite form this statement is contained in [14] (the paragraph before Eq.(2.4))

Let us make the final outlook on the discussion.

- The method developed in [2]-[3] and presented in detail in [4]-[7] is basically quantum theory in Heisenberg picture in quasiclassical approximation. So, in use of the theory one have to proceed with operator calculus. In many case, but not always, the final result can be expressed in classical terms (also in a very definite form). Of course, the possibility to use classical form is a great advantage of the method since it simplifies essentially a solution of a particular problem and permits to solve problems which could not be solved by other methods.

- Zakharov [1] expressed pretensions to Eq.(2.1) of [14]. This formula is given as known with the reference to Eq.(9.27) of [4]. So, to use this equation one have to read, at least, the mentioned Sec.9 of this book. The problem was considered in detail also in books [5], [6] which contain derivation of the mentioned formula. However these books were not mentioned in [1] as well as known textbook [7]. Instead of this Zakharov invents his

own prescriptions which have nothing common with our method and then criticizes these prescriptions. See, e.g. p.4, point 2 "...the distribution function... should satisfy the kinetic equation in which the collisional term contains the *classical* scattering cross section".

- It is evident that there is essential difference between the classical theory and the quantum theory in quasiclassical approximation. Unfortunately, from Comment [1] it looks that Zakharov don't distinguish between these two.

- It is curious that any references on incorrect expressions in our publications are absent in Comment [1].

- It is rather strange to read that in 1996 Zakharov in [19] quite successfully derived formulae, in absence of an external field, contained in our paper [14] and book [4]. Moreover, he explains what kind of cross section should be substituted (just the formulae used in the mentioned publications).

- It should be noted that although Zakharov reproduced some of our results many years later than they were obtained in original papers, he never refer to our publications before and only in Comment [1] there is citing of our publications.

- It's amusing that in list of criticized papers there is paper [20] devoted to quantum theory of the transition radiation where there is no scattering at all (the criticism devoted to description of scattering) and this once more shows the level of the pretensions.

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